Correspondence

Fast Sweep Measurements of Relaxation Times in Superconducting Cavities

Conventional equilibrium methods used to determine the quality of cavity resonators [1] become quite inaccurate if cavities with a high Q, especially superconducting cavities with $Q \sim 10^6$ or more, are to be measured. A static measurement of the cavity impedance requires an extremely small frequency drift, and swept frequency methods require a very slow variation of the frequency because of the long energy relaxation times involved. If random frequency fluctuations due to generator noise are comparable to the bandwidth of the cavity $\Delta\omega_R$, measurements of half-power widths become inaccurate.

Typical 'noisy' resonance curves of a superconducting cavity oscillating in a TE_{111} mode at 9375 and at 9377 Mc/s, as recorded with a slow frequency sweep, are shown in Fig. 1. The cylindrical cavity is made of copper, plated electrolytically with a lead layer. It is coupled through a circular iris to a rectangular waveguide. The resonance curves represent the reflected power with the cavity immersed in liquid helium at 4.2 °K. The two resonances, separated by 2 Mc/s, are caused by a slight mechanical asymmetry, which splits the angular degeneracy of modes in a circular cylinder. Fig. 1 clearly shows the perturbation caused by FM noise in the X13 klystron, which is used as an RF generator. The resonator accepts a narrow portion of the noise spectrum centered about its resonant frequency ω_R. If the signal frequency $\omega(t)$ is close to ω_R, the filtered noise causes a more or less periodic perturbation, with a frequency commensurate with the half-power width $\Delta\omega_R$ of the cavity [2].

Frequency drift and noise interference can be avoided by using relaxation methods to determine Q values of high quality cavities. A simple scheme is based on the conventional frequency sweep method. If the rate of frequency sweep $\Delta\omega/T$ is increased so that frequency variations comparable to the half-power width occur within the relaxation time τ of the cavity, i.e., if

$$\frac{\Delta\omega}{T} \sim \frac{\Delta\omega_R}{\tau} = (\Delta\omega_R)^2$$

one observes a characteristic distortion of the resonance curve. The energy stored in the cavity during the passage of the signal frequency through its spectral bandwidth relaxes in the form of a damped oscillation at the resonance frequency ω_R. The superposition of this transient oscillation ω_R on the instantaneous generator frequency $\omega(t)$ produces a beat frequency in the nonlinear detector, provided the beat is fast enough. The beat signal has been observed with the superconducting cavity and is shown in Fig. 2 for two different sweep rates. The beat frequency increases with time according

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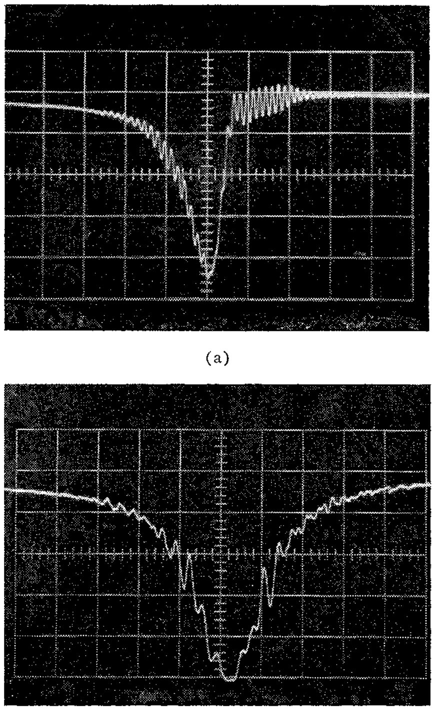
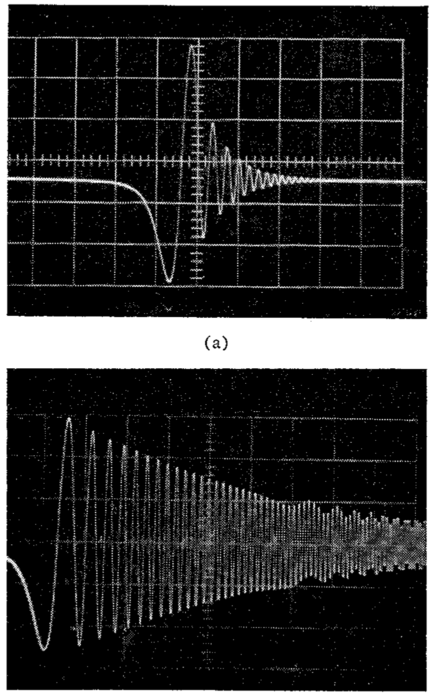


(a)

(b)

Fig. 1. Resonant curves of quasi-degenerate TE_{111} modes in a superconducting cavity, measured in reflection with slow frequency sweep. (a) $f_R = 9375$ Mc/s, time scale $= 0.4$ µs/div, sweep rate $= 50$ Mc/s². (b) $f_R' = 9377$ Mc/s, time scale $= 2$ µs/div, sweep rate $= 0.5$ Mc/s².



(a)

(b)

Fig. 2. Beat phenomenon (Cady's Click) between transient ringing of a superconducting cavity at 9375 Mc/s and swept frequency for different sweep rates. (a) time scale $= 40$ µs/div, sweep rate $= 1.33 \cdot 10^3$ Mc/s². (b) time scale $= 4$ µs/div, sweep rate $= 8 \cdot 10^4$ Mc/s². (Secondary interference is caused by quasi-degenerate mode at 9377 Mc/s.)

to a linear frequency sweep used in this experiment. In acoustics, this beat phenomenon can be observed on piezoelectric resonators. It is often audible and known as "Cady's Click" [3].

In order to establish the amplitude and the decay rate of the beat phenomenon, we have considered a simple equivalent series circuit with an inductance L, a capacitance C, and a resistance R, driven by a voltage

$$V = \begin{cases} 0 & t < t_0 \\ e^{j\psi} & t \geq t_0 \end{cases} \tag{1}$$

with a time-varying frequency

$$\omega(t) = \frac{\partial\psi}{\partial t} = \omega_R + \frac{\Delta\omega}{T}\,t. \tag{2}$$

Here

$$\omega_R = \frac{1}{\sqrt{LC - \left(\frac{4R}{LC}\right)^2}}$$

denotes the resonant frequency. By convolution of the impulse response one obtains for the current in the circuit

$$I(t) = \frac{1}{L}\int_{t_0}^{t} e^{j\omega_R x + j(\Delta\omega/2T)x^2} \cdot \left[e^{-(R/2L)(t-x)} \left\{ \cos\omega(t-x) - \frac{R/2L}{\omega_R}\sin\omega(t-x) \right\} \right] dx. \tag{3}$$

The integral may be solved rigorously in terms of error functions. However, the two important limiting cases corresponding to very slow frequency variation and to very fast frequency variation can be obtained directly from (3). For a slow sweep, i.e., with $\Delta\omega/T \ll (\Delta\omega_R)^2$, a substitution $t - x = \xi$ yields the quasi-static resonance curve, if terms in ξ^2 are neglected in the exponent. For a fast sweep with $\Delta\omega/T \gg (\Delta\omega_R)^2$, the response is obtained by evaluation of the contribution from the vicinity of the stationary phase point [4] at $x = 0$,

$$I(t) \sim \left(1 - \frac{R/2L}{j\omega_R}\right)\sqrt{\frac{2\pi}{(\Delta\omega/T)}}\,\frac{1}{2L} \cdot e^{-(R/2L)t} e^{j(\omega_R t + \pi/4)} \sim I_m e^{-(R/2L)t} e^{j(\omega_R t + \pi/4)}. \tag{4}$$

This expression is valid for times $t \gg (T/\Delta\omega)\Delta\omega_R$. It shows a damped ringing of the cavity at its resonant frequency ω_R after excitation by a signal whose frequency passes quickly through the bandwidth of the cavity. The excitation is seen to decrease with an increasing sweep rate $\Delta\omega/T$.

At the nonlinear detector diode, mixing of the swept frequency wave and the cavity ringing gives rise to the beat phenomenon.[1]

[1] For transmission type cavities a part of the incident signal may be fed separately to the detector.

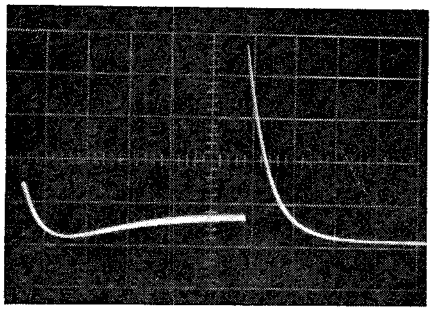


Fig. 3. Reflection of RF pulse from a superconducting cavity. Overcoupled mode, pulse carrier frequency $=f_R=9375$ Mc/s, pulse duration 110 μs, time scale 20 μs/div. Every relaxation time is read from decay after termination of pulse.

For a sufficiently fast frequency sweep, the amplitude of the transient ringing is small compared to the amplitude of the carrier signal I_0, $m=(I_m/I_0)\ll 1$. Neglecting higher powers of m, one obtains for the detector current

$$i \sim \text{const}\left[1 + 2me^{-(R/2L)t} \cdot \cos\left(\frac{\Delta\omega}{2T}t^2 + \frac{\pi}{4}\right)\right]. \quad (5)$$

Equation 5 shows that the decay of the beat signal is a direct measure of the loaded quality of the cavity

$$\tau_{\text{ampl}} = \frac{2L}{R} = \frac{2Q_L}{\omega_0} = 2\tau.$$

Since a decay proportional to the field amplitude is measured, errors caused by nonquadratic characteristics of the diode are reduced. Also, because of the high sweep rate the measurement is insensitive to spurious FM noise. However, the response time of the demodulator and of the oscilloscope must be fast enough to resolve the beat signal. More precisely, the upper cutoff frequency of the detector must be high compared to the beat frequency when the beat amplitude has decreased by a factor $1/e$, hence,

$$\omega_{\text{cutoff}} > 2\frac{\Delta\omega}{T}\tau.$$

If the response of the receiving system decreases towards higher frequencies, the measured value of the time constant is too small.

The decay time of the beat signal (Fig. 2) has been compared with other nonequilibrium measurements using RF pulses with rise and fall times which are short compared to τ [5]. Figure 3 shows the time behaviour of RF pulses after reflection from the cavity. The carrier frequency has been adjusted to the resonant frequency of the cavity. The measured relaxation time after termination of the pulse is 10 μs, compared to an amplitude decay time of 20 μs observed with rapid frequency alteration and response. The loaded Q of the resonator in the superconducting state is $Q_L=\omega_R\tau \sim 5.9\cdot 10^5$ for the strongly excited resonance (overcritically coupled) and $Q_L \sim 6.3\cdot 10^6$ for the weakly excited quasi-degenerate mode (undercritically coupled). The half-power widths calculated from the decay times are $\Delta f=15.9$ kc/s and $\Delta f=1.5$ kc/s, respectively, in fair agreement with the values of Δf extrapolated from the quasi-static resonance curve.

It is interesting to note that with a sufficiently fast frequency modulation the beat phenomenon can be observed in any conventional microwave cavity. Calculations show that with an increasing sweep rate the resonance curve first becomes asymmetrical with a smooth initial flank and a gradually increasing overshoot. The apparent width of the resonance curve is increased, i.e., with a fast sweep the Q value determined from the half-power points is too low.

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References

[1] A. F. Harvey, *Microwave Engineering*. New York: Academic, 1963, pp. 202–204.
[2] S. O. Rice, "Statistical properties of a sine wave plus random noise," *B.S.T.J.*, vol. 27, pp. 109–157, January 1948.
[3] W. G. Cady, *Piezoelectricity*, vol. 1. New York: Dover, 1964, pp. 385–387.
[4] A. Erdelyi, *Asymptotic Expansions*. New York: Dover, 1956.
[5] H. Zimmer, "Ein supraleitender Resonator hoher Güte bei 9375 MHz," *Philips: Unsere Forschung in Deutschland*, Philips Zentrallaboratorium, GmbH, Aachen, Hamburg, 1964, pp. 134–136.

Re-Entrant Directional Couplers Having Direct Coupled Center Conductors

The re-entrant coaxial coupler [1] and its strip transmission line equivalent [2] are shown in Figs. 1 and 2, respectively. In both configurations, no direct coupling between center conductors exist. The purpose of this communication is to point out that if direct coupling between center conductors is permitted, the required value of Z_{01} is reduced [1] (see Figs. 1 and 2), thus often permitting easier realization of the coupler than would otherwise be the case.

For both the networks of Figs. 1 and 2, the capacitance network at an arbitrary cross section of the re-entrant coupler with direct coupling between center conductors is given in Fig. 3. The direct capacitance between center conductors is C_{12}. The capacitance between one center conductor and the floating shield is C_{N1}. The capacitance between the shield and ground is C_{NG}.

The design equations for the network of Fig. 3 are

$$376.7\,Y_{oo}/\sqrt{\epsilon_r} = C_{N1}/\epsilon + 2C_{12}/\epsilon \quad (1)$$

and

$$\frac{\sqrt{\epsilon_r}\,Z_{oe}}{376.7} = (C_{N1}/\epsilon)^{-1} + 2(C_{NG}/\epsilon)^{-1} \quad (2)$$

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where:

Z_{oe} = the required even-mode impedance of the directional coupler [3]
Y_{oo} = the required odd-mode admittance of the directional coupler [3]
ϵ_r = the relative dielectric constant of the medium, and
ϵ = the permittivity of the medium.

In general,

$$C/\epsilon = \frac{376.7}{\sqrt{\epsilon_r}\,Z} = \frac{376.7\,Y}{\sqrt{\epsilon_r}}. \quad (3)$$

Note that for $C_{12}=0$ (1) and (2) reduce to (1) and (2) of Cohn [1].

The normalized capacitance C_{NG}/ϵ is determined from the network of Fig. 4 and by (3). In this case the center conductors are driven in the even-mode [3]. The normalized capacitances C_{12}/ϵ and C_{N1}/ϵ are determined from the network of Fig. 5. In this case

$$C_{N1}/\epsilon = 376.7\,Y_{oe}/\sqrt{\epsilon_r} \quad (4)$$

and

$$C_{12}/\epsilon = \frac{376.7}{\sqrt{\epsilon_r}}\,\frac{(Y_{oo}-Y_{oe})}{2}, \quad (5)$$

where Y_{oo} and Y_{oe} are the odd- and even-mode admittances [3], respectively, of the network of Fig. 5.

A short numerical example will show how direct coupling between center conductors reduces the required Z_{01}. Let it be required to design a -3.0 ± 0.4-dB directional coupler over a 1.93-to-1 frequency band. Let the relative dielectric constant of the medium be 2.3. The required coefficient of coupling is

$$k^2 = 0.55. \quad (6)$$

It is determined from design equations given in [3] that

$$\sqrt{\epsilon_r}\,Z_{oe} = 197.7 \quad (7)$$

and

$$\sqrt{\epsilon_r}\,Z_{oo} = 29.33. \quad (8)$$

From (1) and (2), the required normalized capacitances satisfy

$$0.5248 = (C_{N1}/\epsilon)^{-1} + 2(C_{NG}/\epsilon)^{-1} \quad (9)$$

and

$$12.84 = C_{N1}/\epsilon + 2C_{12}/\epsilon. \quad (10)$$

Case 1: $C_{12}/\epsilon=0$. With $C_{12}=0$, (9) and (10) may be solved directly.

$$C_{N1}/\epsilon = 12.84 \quad (11)$$

and

$$C_{NG}/\epsilon = 4.475. \quad (12)$$

Equation (12) corresponds to

$$\sqrt{\epsilon_r}\,Z_{01} = 84.2, \quad (13)$$

where we recall that Z_{01} is determined from the network of Fig. 4.

Case 2: $C_{12}/\epsilon=1.5$. Then

$$C_{N1}/\epsilon = 9.843, \quad (14)$$

and

$$C_{NG}/\epsilon = 4.726. \quad (15)$$